Pakistan Journal of Hydrocarbon Research
Vol. 8, No. 1 (January 1996). p. 63-71. 22 Figs 1 Table

A Critical Comparison of Differentiation Algorithms for Transient Well Test Analysis

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ABSTRACT

Pressure derivative curves are used as a powerful diagnostic tool in pressure transient well test analysis. In this paper the performance of various differentiation algorithms has been evaluated. Various smoothing procedures to be used in conjunction with differentiation algorithms are also discussed. Finally the best combination of smoothing and differentiation algorithms has been recommended.

INTRODUCTION

Pressure derivative approach is quite well known as a tool which greatly simplifies the pressure transient well test data interpretation process. One of the main problems inherent with the pressure derivative approach is that the rate of change of pressure, the quantity under consideration, currently cannot be measured directly and must be extracted from discrete measurements of the absolute pressure evolution. The numerical techniques called differentiation algorithms, are used to evaluate the derivative of the pressure /time data. These discrete pressure measurements have superimposed upon them random errors which, regardless of their source, are characteristically described as noise. The noise in pressure / time data makes the differentiation difficult, if not inconclusive. An attempt to improve the signal to noise ratio can also distort the signal trend. An ideal differentiation algorithm must have the ability, besides differentiation, to suppress the noise (or smoothing) on one side and the degree of data distortion on the other.

Several algorithms (Bourdet et al.,1983, 1989 and Clark et al., 1985) have been proposed to obtain the derivative of pressure/time data. In these studies the authors have presented brief descriptions of the selected algorithms and their conclusions. None of them presented the results upon which their conclusions were based. This missing detail was the main initiative to undertake work presented in this paper. The objective of this paper is to evaluate the performance of available techniques for differentiation and smoothing. It also includes few techniques which were not introduced by the above mentioned authors.

Performance of a differentiation algorithm is evaluated by measuring deviation of computed derivative from the true derivative of the data. If actual data is used as an example data, its true derivative is not known. Therefore a modified type curve of Agarwal et al. (1970) was used to evaluate different methods.

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PRELIMINARIES

The term "Pressure Derivative" is a misleading one. In this section first we will explain the background of this term. Agarwal et al. (1970) presented a mathematical model for liquid flow in homogenous porous media towards a well with constant wellbore storage and skin. They related dimensionless pressure drop (P_{wd}) with skin (S), dimensionless wellbore storage (C_d), and dimensionless time (t_d) as follows:

$$P_{wd}=(4/\pi^2)[1-\exp(-u^2t_d)]du \,/\, u^3\{[uC_dJ_0-(1-C_dSu^2)J_1]^2 + [uC_dY_0-(1-C_dSu^2)Y_1]^2\} \quad (1)$$

They also presented a relationship for derivative of P_{wd} with respect to t_d as follows;

$$dP_{wd}/dt_d=(4/\pi^2)\exp(-u^2t_d)\, du \,/\, u\{[uC_dJ_0-(1-C_dSu^2)J_1]^2 +[uC_dY_0-(1-C_dSu^2)Y_1]^2\} \quad (2)$$

Where J_n and Y_n are the Bessel functions of the first and second kind of nth order.

Gringarton et al. (1979) presented a type curve which consists of Pwd, from equation (1), versus t_d/C_d, each curve being characterized by a value of C_de^{2S}. One of those curve is shown in figure 1. Bourdet et al. (1983) took slope of Gringarton type curve and called it pressure derivative, as follows;

$$\begin{aligned} \text{slope} &= dP_{wd}/d[\ln (t_d/C_d)] \\ &= (t_d/C_d)\, dP_{wd}/d(t_d/C_d) \\ &= t_d\, dP_{wd}/dt_d \end{aligned} \quad (3)$$

It is infact a semilog derivative. The derivative from equation (2) is called "primary pressure derivative" (Matter and Zaoral, 1992). Bourdet et al. (1983) presented plots of pressure derivative versus t_d/C_d as "derivative type curves". The derivative curve corresponding to the selected dimensionless pressure curve is also shown in figure 1.

Appendix 1 presents the example data and resulting relationships used to describe the selected curves in terms of real parameters. The resulting curves are shown in Figure 2. Then noise was introduced into pressure drop curve by shifting its value at 6.7 hours from 2876.952 psi to 3040 psi (see Figure 3). This noised data was used as an example data to test various differentiation algorithms. The criterion of comparison consists of following three checks:

Degree of scatter

The algorithm produces in the derivative curve computed from noised data. The degree of scatter is also an indirect measurement of the degree of smoothing i.e., larger is the degree of scattering, lesser is the smoothing. A measure of degree of scatter can be obtained by the following error formula.

$$\text{Error} = \Sigma \text{ [abs(ideal derivative value - noised derivative value)/ ideal derivative value]} \quad (4)$$

Shape distortion

The algorithm produces compared to the shape of ideal derivative curve. A measure of the shape distortion is horizontal or vertical shift between the two derivative curves.

Time taken by the algorithm

Discussion and the analysis done in this paper is divided into three parts. In part A, algorithms for obtaining pressure derivative from the above mentioned noised data will be discussed and the best one will be selected to be used in part B. In part B, the effect of smoothing algorithms used along with differentiation algorithm is discussed. Part C is similar to Part B except that the example data is now a real one instead of a noised type curve.

DISCUSSION (A)

Weighted Average of Slopes

Bourdet et al. (1983) recommended this algorithm. It uses one point before and one point after the point of interest, calculates the corresponding derivatives, and places their mean at the point considered.

$$(dP/dX)_i = [(P_i-P_1)(X_2-X_i) / (X_i-X_1) + (P_2-P_i)(X_i-X_1) / (X_2-X_i)] / [(X_i-X_1)+(X_2-X_i)] \quad (5)$$

where

- P = p
- 1 = point before
- 2 = point after
- X = time function
  - = ln t for drawdown data
  - = ln (Horner time) for buildup data

Figure 4 shows the ideal derivative data and derivatives obtained using this methods for the noised data. It can be noticed from the figure that the shape is preserved except at the points adjascent to the noised data point. The degree of freedom is n-2 where n is number of pressure points. Cumulative error came out to be 18.83%. On a DX 486 processor, this algorithm took 0.2734 seconds.

Slope of Straight Line

Clark et al. (1985) mentioned this method. This is a least square procedure. It selects a region around the point to be considered and fits a straight line through the points in the region. The slope of the resulting straight line is considered as the pressure derivative at the point under consideration which is the middle point of the region. Figures 5 to 7 show the results for 3, 5 and 7 point regions respectively. It can be seen from these figures that size of region does not effect the shape of the derivative curve except the number of derivative points scattered due to noised pressure point. The number of scattered derivative points increased with increase in size of the region. The cumulative error varied inversely to size of the region. It means even though the number of scattered points increased with increase in size of the region but the degree of scatter decreased. Maximum error was 18.83 % while using 3 point region, which is same as the error found in method 1. The degree of freedom was n-m+1 where m is number of points in the region. Time taken by this algorithm increased with the size of the region. The minimum time taken was 0.2695 seconds using 3 point region. Detail results for each region are presented in Table 1.

Slope of Parabola

Bourdet et al. (1989) described this technique. This is another least square method which selects a region around the point to be considered and fits a parabola through the points in the region. Then exact derivative of the parabola is used to find the derivative of the point under consideration. Figures 8 to 10 show the results for 3, 5 and 7 point regions respectively. It can be seen from the figures that size of region does effect the shape of the derivative curve. The distortion in the shape of derivative curve increase with increase in the size of the region except the number of derivative points scattered due to noised pressure point. The number of scattered derivative points increased with increase in size of the region. The cumulative error varied directly proportional to the size of the region. Minimum error was 18.86 % while using 3 point region. The degree of freedom was n-m+1 where m is number of points in the region. Time taken by this algorithm increased with the size of the region. The minimum time taken was 3.4589 seconds using 3 point region. Detailed results for each region arepresented in Table 1.

FIGURE 1: IDEAL DRAWDOWN TEST

DIMENSIONLESS UNITS

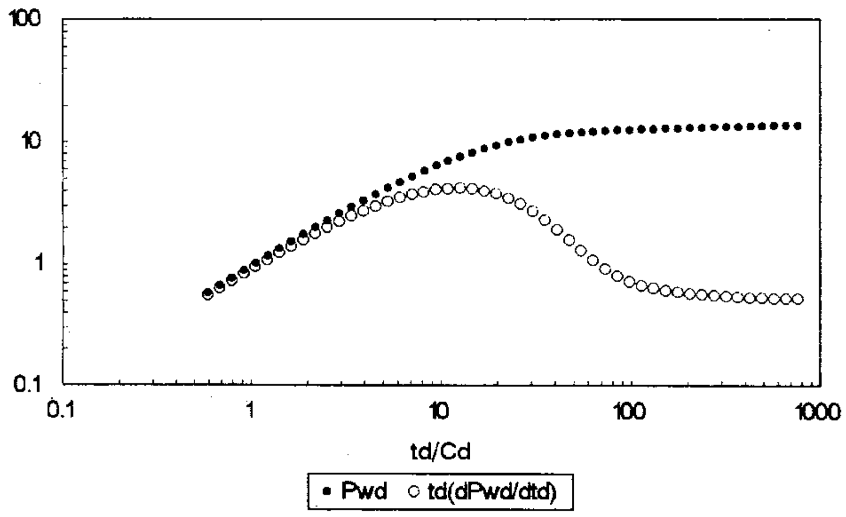


FIGURE 2: IDEAL DRAWDOWN TEST

IN OIL FIELD UNITS

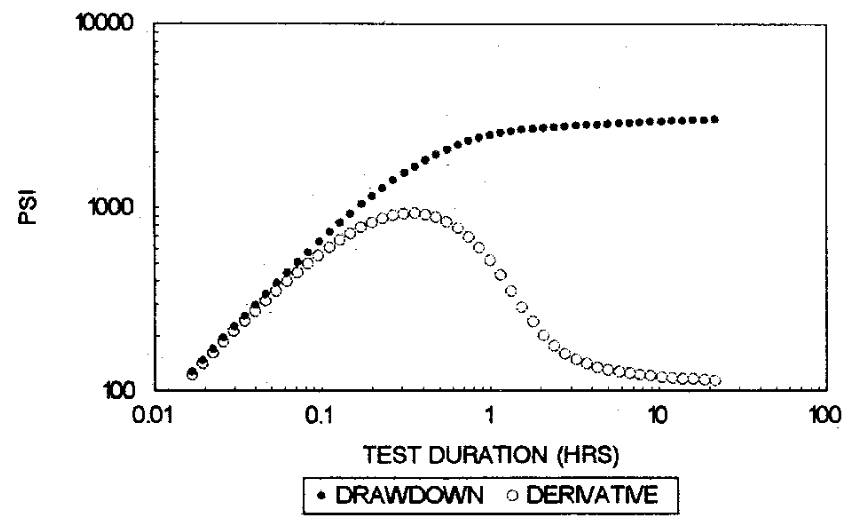


FIGURE 3: IDEAL DATA

WITH SINGLE ISOLATED ERROR POINT

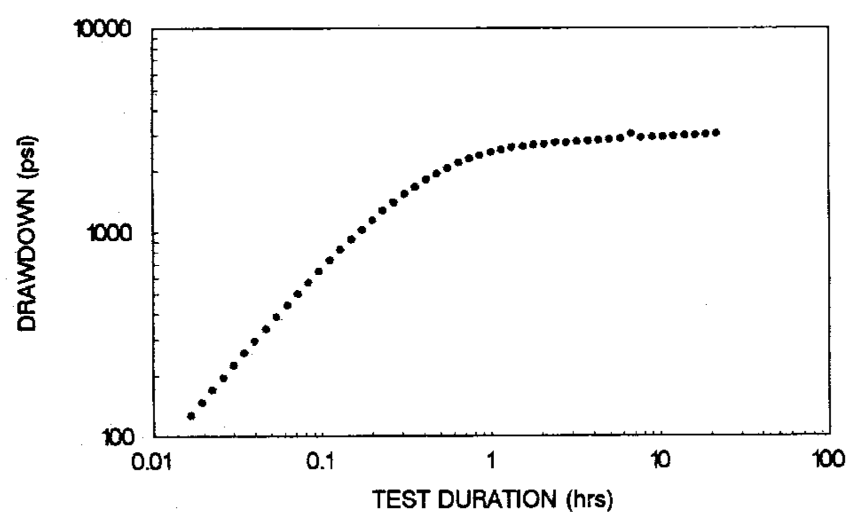


FIGURE 4: METHOD 1

WEIGHTED AVERAGE OF SLOPES

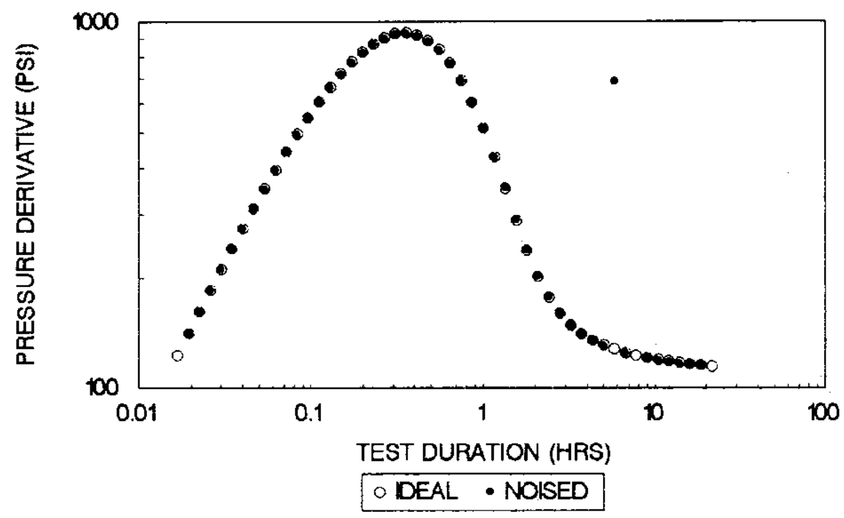


FIGURE 5: METHOD 2

SLOPE OF STRAIGHT LINE
USING 3 POINTS

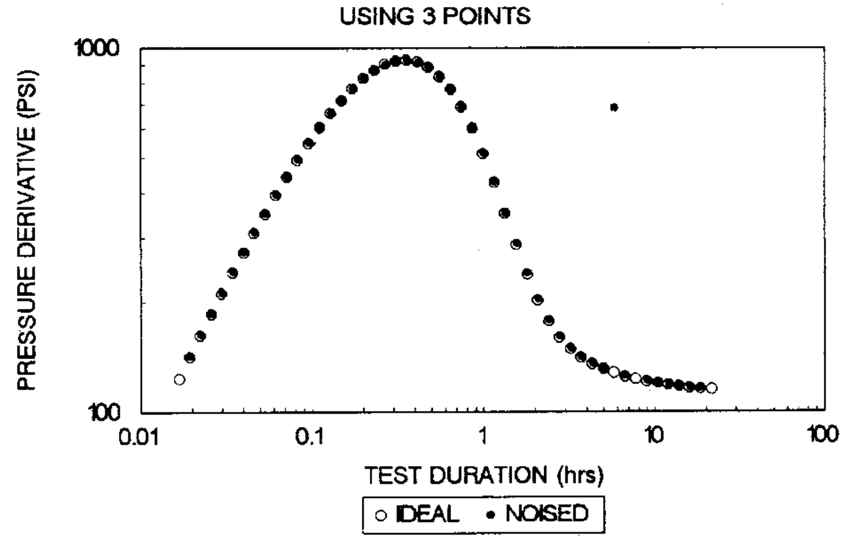


FIGURE 6: METHOD 2

SLOPE OF STRAIGHT LINE
USING 5 POINTS

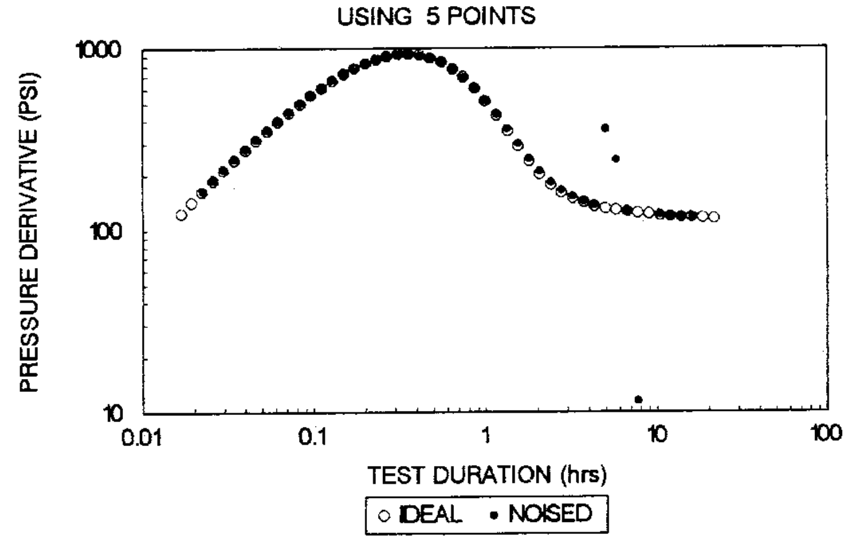


Table 1. Summary of the results.

METHOD	ERROR (%)	SHAPE	TIME (sec)	NOISE HANDLING	SCATTERED POINT	DEGREE OF FREEDOM
Weighted Average of Slopes	18.83	undisturbed	0.2734	ineffective	2	n-2
Slope of Straight line (3 point)	18.83	undisturbed	0.2695	ineffective	2	n-2
(5 point)	12.43	undisturbed	0.3281	ineffective	4	n-4
(7 point)	10.33	undisturbed	0.3789	ineffective	6	n-6
Slope of Parabola (3 point)	18.86	undisturbed	3.4589	ineffective	2	n-2
(5 point)	30.89	disturbed	3.9511	ineffective	5	n-4
(7 point)	39.36	disturbed	4.4492	ineffective	7	n-6
Orthogonal Polynomials (5 point)	29.09	undisturbed	0.2187	ineffective	4	n-4
(7 point)	23.59	undisturbed	0.2695	ineffective	6	n-6
(9 point)	26.39	disturbed	0.2695	ineffective	9	n-8

Orthogonal Polynomial Method

Savitzky and Golay (1964) introduced the application of orthogonal polynomials in data smoothing and differentiation. This is basically simplified least square procedure. A region around the point to be considered is selected and the ordinate values of the points in the region are convolved with standard set of integers called convoluting integers. To perform a convolution of the ordinate numbers of the points in the region with a set of convoluting integers, each number in the set is multiplied by the corresponding number in the region, the resulting products are added and this sum is divided by a number called normalizing factor. There are two important restrictions for the application of this method. First the points must be at a fixed uniform interval in the chosen abscissa. Second the curves formed by graphing the points must be continuous and more or less smooth. Figures 11-13 show the derivative values computed using this method for 5,7 and 9 point regions respectively. Cumulative error decreased first with enlargement of region from 5 to 7 points but increased again when the region was enlarged further from points 7- 9. Shape got disturbed when using 9 point region. The method takes least time compared the ones required by above mentioned methods. Degree of freedom decreases with increase in size of the region. The number of scattered points increases with the increase in size of the region. So like other previously mentioned techniques, this method was also proved ineffective in handling noise in the pressure data.

ANALYSIS (A)

Table 1 summarizes the results discussed above. It can be noticed from this table that non of the algorithms discussed removes the noise while computing derivative. Methods of fitting parabola and orthogonal polynomials can be rejected because they disturb the shape of the derivative curve. Method 1 and 2 do not disturb the shape of the derivative curve. Considering number of scattered points, as another check, leaves us with the method 1 and the method 2 with 3 point region. They carry same accuracy. Method 2 with 3 point region is slightly quicker but method 1 offers the ease of calculation even without a computer. It leads us to select the method 1 as the best available among the ones discussed. Signal to noise ratio must be improved before we take derivative of the signals, because as we have observed, differentiation algorithm does not do it. Therefore, in section B various smoothing procedures are discussed in conjunction with the method 1.

DISCUSSION (B)

Method of non-Adjascent Points

As shown in Equation 3, the pressure derivative is a function of ratio of change in pressure drop and corresponding time interval. Modern electronic gauges record pressure points at high sampling rate (reading every few seconds) i.e., very small time intervals. So even a little noise superimposed on the signal will be magnified during differentiation because of denominator . Bourdet et al. (1989), suggested to choose the points, where the derivative is calculated, sufficiently distant from the point under consideration. It increases the time interval which causes less magnification of noise. It also increases the corresponding pressure drop which makes the noise less effective. The minimum distance, (L), considered between the abscissa of the points and the point under consideration is expressed in terms of the time function , X. This methods selects points 1 and 2 as being the first ones such that $\Delta X_{1,2}$ L. For present example which is a draw down test,

$$\Delta X_{1,2} = \log t_2 - \log t_1$$

Common values for L are 0 (consecutive points) up to 0.5 in extreme cases. Using L=0.2 the results are shown in Figure 14. Cumulative error was decreased to 10.6% from 18.88 %.

Method of Orthogonal Polynomials

Savitzky and Golay (1964) also presented a technique for data smoothing or to improve signal to noise ratio. This work is very similar to their technique for data differentiation. They provided the values of convoluting integers and normalizing functions for various region sizes. Figure 15 shows the results of using this smoothing technique for 5 point region Cumulative error increased from 18.8% to 20.4%.

Method of dirty point filtering

Khan (1994) used the dirty point filtering to filter noise from geophysical signals. The term 'dirty point' is equivalent to the term 'noised point' which means sudden fluctuation of data which break the continuum of data trend. Each point is judged, according to given criteria, for its noise level. If it is found dirty then the point is discarded from the data before differentiation. For a point (x_i, y_i) the two consecutive slopes are found

$$A = (y_i - y_{i-1})/(x_i - x_{i-1})$$
$$B = (y_{i+1} - y_i)/(x_{i+1} - x_i)$$

Then an average of consecutive ordinates is found

$$y_{avg} = 0.5\,(y_{i-1} + y_{i+1})$$

$$CC = h\ y_{avg}$$

The factor 'h' is a fluctuation limiting variable and its suitable value is found by iteration. Then a dirty point is found if following two conditions are satisfied.

I) A and B are complementary to each other

either $A>0$ and $B<0$
or $A<0$ and $B>0$

II) either $y_i > y_{avg} + CC$
or $y_i < y_{avg} - CC$

Using this method for smoothing the data before differentiation, the results are presented in Figure 16. The elimination of noised point reduced the cumulative error from 18.83% to 0.2%.

Method of Moving Arithmetic Average

In this method the points in the selected region are added and the sum is divided by the number of the points. Figure 17 shows the result of using this method for 5 point region. The error was reduced from 18.83% to 9.76%.

ANALYSIS (B)

Method of dirty point filtering seemed to be the best one because its error is minimum as compared with the other methods. Method of moving arithmetic average and of non-adjascent points are second and third best respectively. The method of orthogonal polynomials had to be rejected because it increased the error. Method of non-adjascent also slightly disturbed the shape of the derivative curve. In the present case it slightly compressed the curve.

FIGURE 7: METHOD 2

SLOPE OF STRAIGHT LINE
USING 7 POINTS

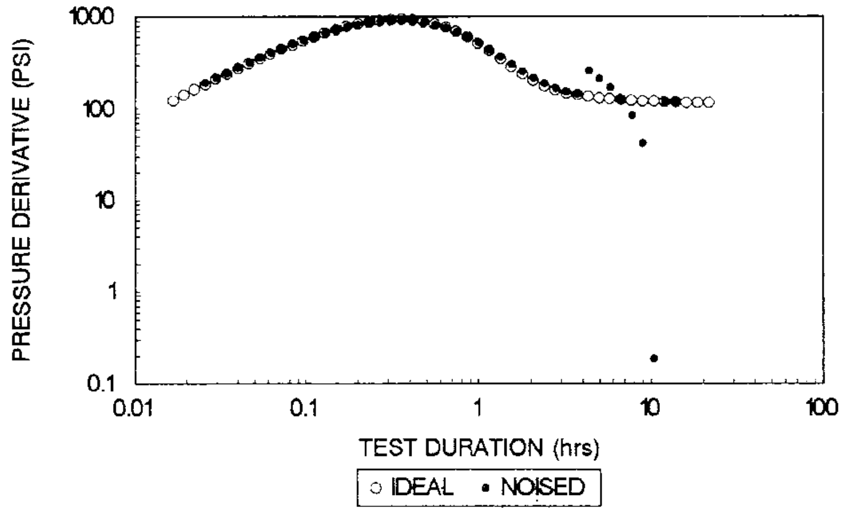


FIGURE 8: METHOD 3

SLOPE OF PARABOLA
USING 3 POINTS

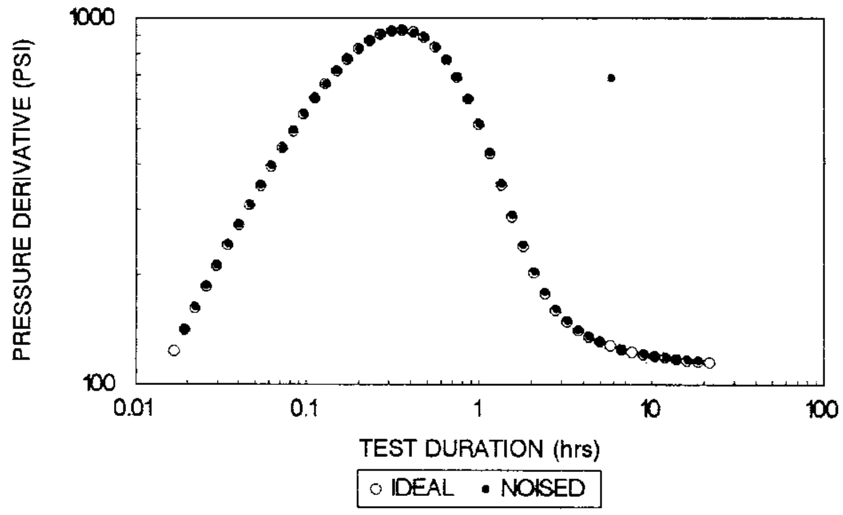


FIGURE 9: METHOD 3

SLOPE OF PARABOLA
USING 5 POINTS

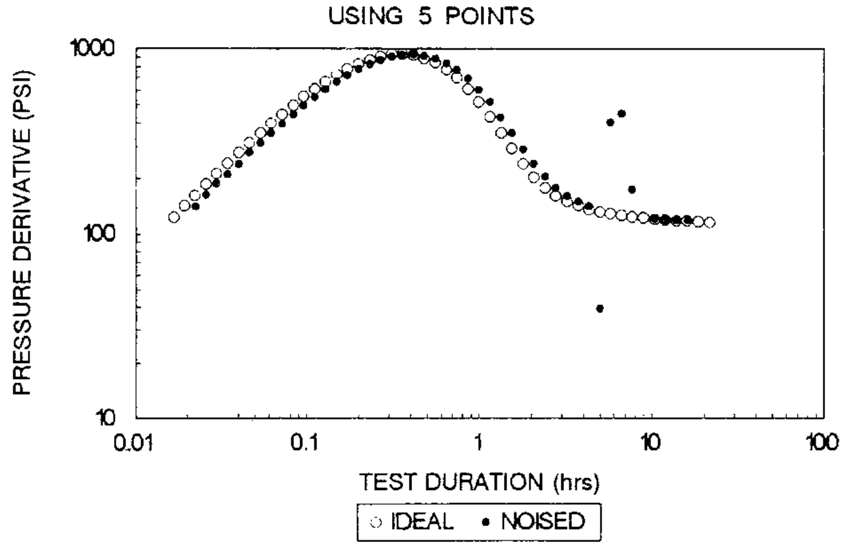


FIGURE 10: METHOD 3

SLOPE OF PARABOLA
USING 7 POINTS

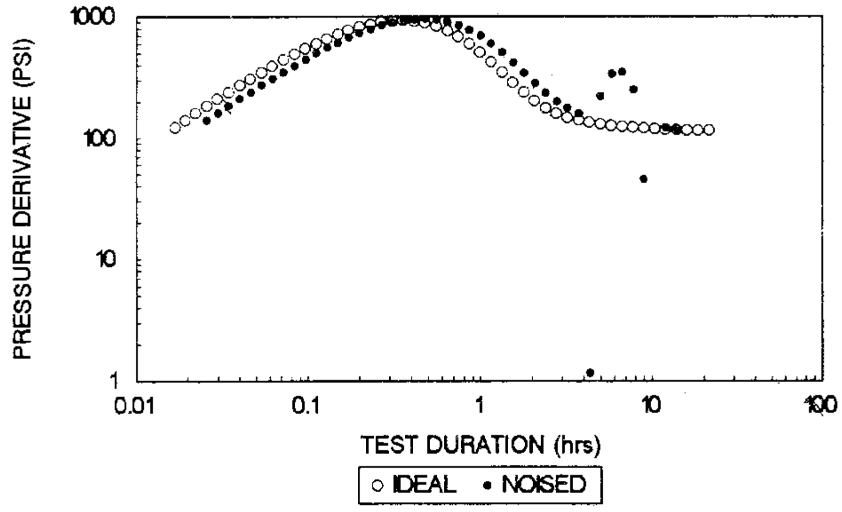


FIGURE 11: METHOD 4

ORTHOGONAL POLYNOMIAL FITTING
USING 5 POINTS

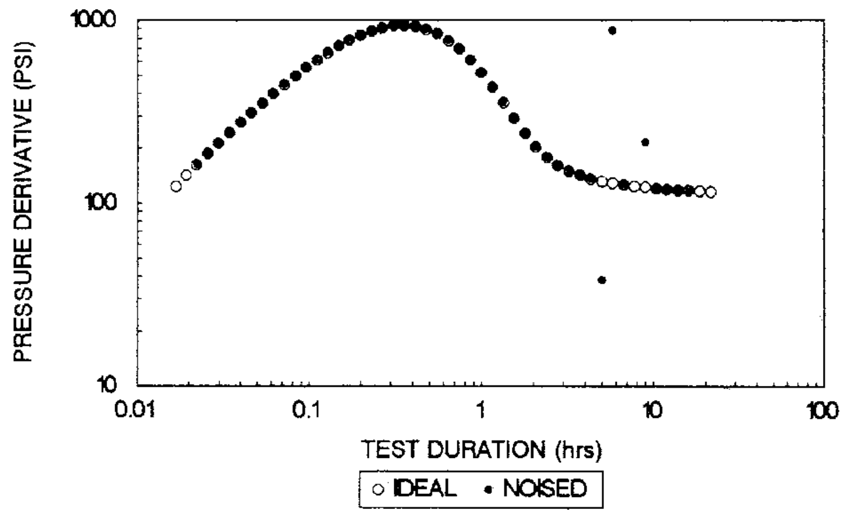
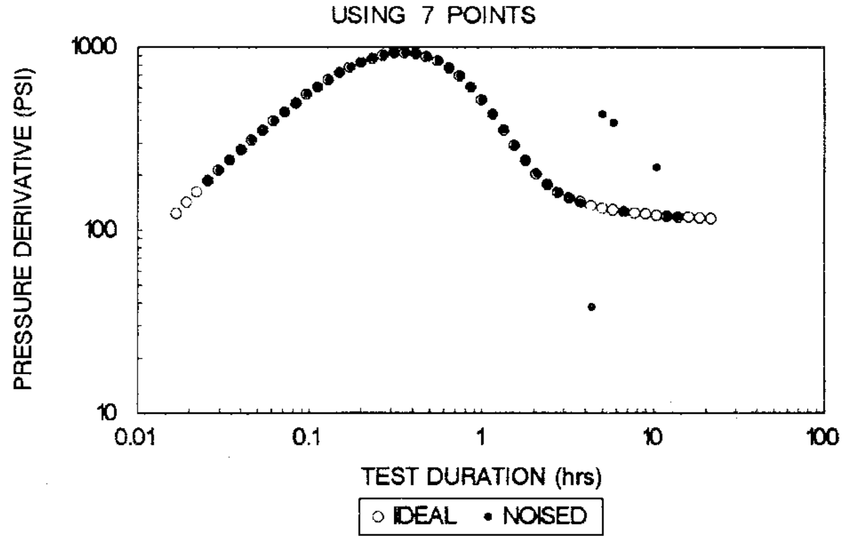


FIGURE 12: METHOD 4

ORTHOGONAL POLYNOMIAL FITTING
USING 7 POINTS



DISCUSSION AND ANALYSIS (C)

Next the selected smoothing algorithm and differentiation were applied to a drawdown data (Horne, 1990) containing multiple noised points and the results are presented in Figure 18-20. The dirty point filtering (Figure 18) ,which perforemed best for single noised point, now performed worst (only 0.4% smoothing). It performs a bit better if applied after differentiation (Figure 21). Even then it did not provide a clear picture at the late time data. The method of non-adjascent points (Figure 20) performed better (10% smoothing) except at the late time data. Now it stood second best instead of its third best which was its relative performance when using single noised point. Using arithmetic average (see Figure 19) relatively maximum smoothing (17.04%) was obtained. The shape was also preserved. So this method performed best compared to its performance as second best when using single noised point. This method provided similar result when applied after differentiation (see Figure 22).

CONCLUSIONS

Smoothing pressure data, through arithmetic average and then differentiating it with weighted average of slope method turned out to be the best combination to obtain pressure derivative.

NOMENCLATURE

C	wellbore storage coefficient (stb/psi)
C_t	total system compressibility (/psi)
d	differentiation operator
h	thickness (ft)
J_0	Bessel function of first kind, order zero
J_1	Bessel function of first kind, order one
K	permeability (md)
P_i	initial Pressure (Psia)
P_{wf}	wellbore flowing Pressure
Δp	pressure drop (psi)
q	liquid production rate (stb/d)
r_w	wellbore radius (ft)
S	skin factor
t	time (hours)
Y_0	Bessel function of second kind, order zero
Y_1	Bessel function of second kind, order one
u	variable of integration

GREEK

β	formation volume factor (res vol/std vol)
μ	viscosity (cp)
ϕ	porosity (pore vol/bulk vol)

FIGURE 13: METHOD 4

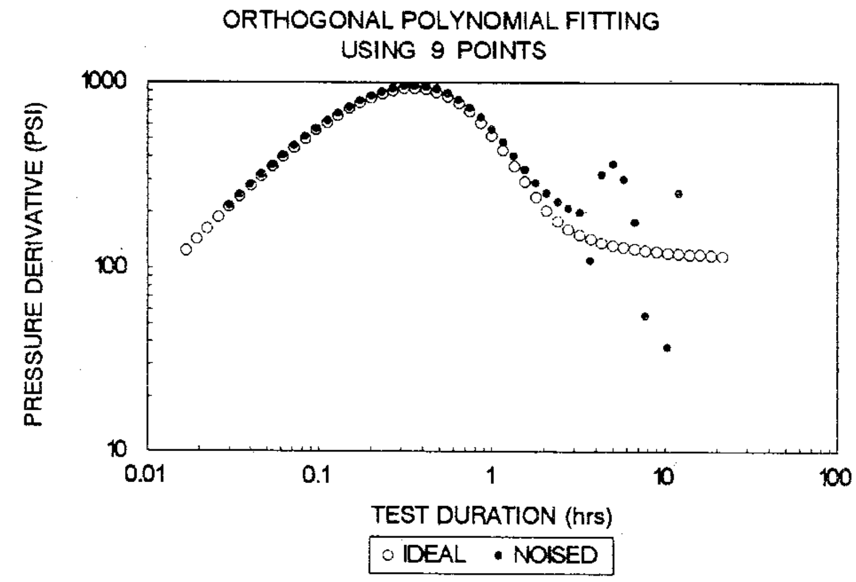


FIGURE 14: SMOOTHING BEFORE DIFFERENTIATION

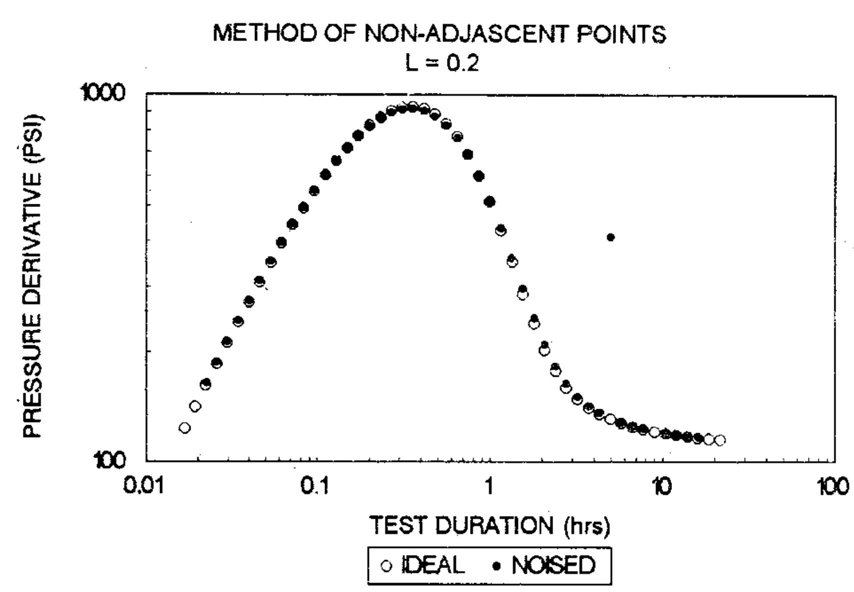


FIGURE 15: SMOOTHING BEFORE DIFFERENTIATION

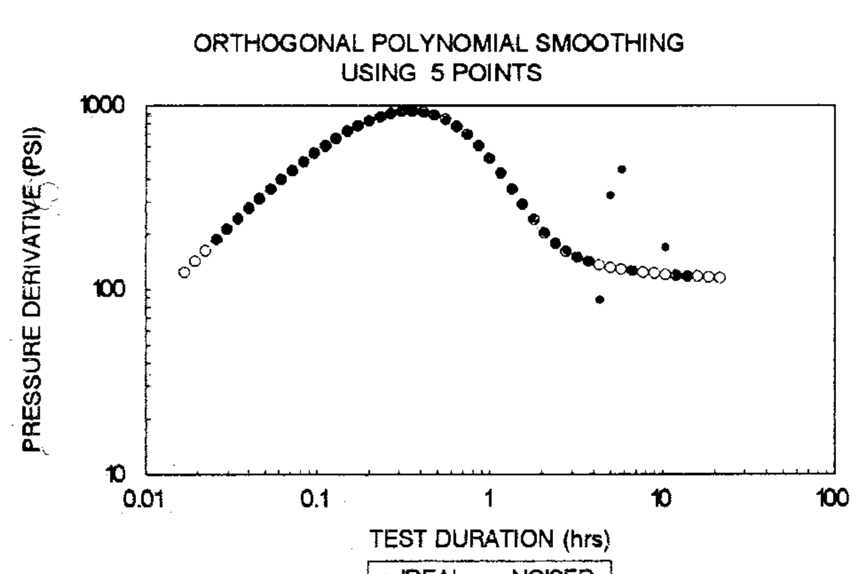


FIGURE 16: SMOOTHING BEFORE DIFFERENTIATION

USING DIRTY POINT FILTERING (h=0.1)

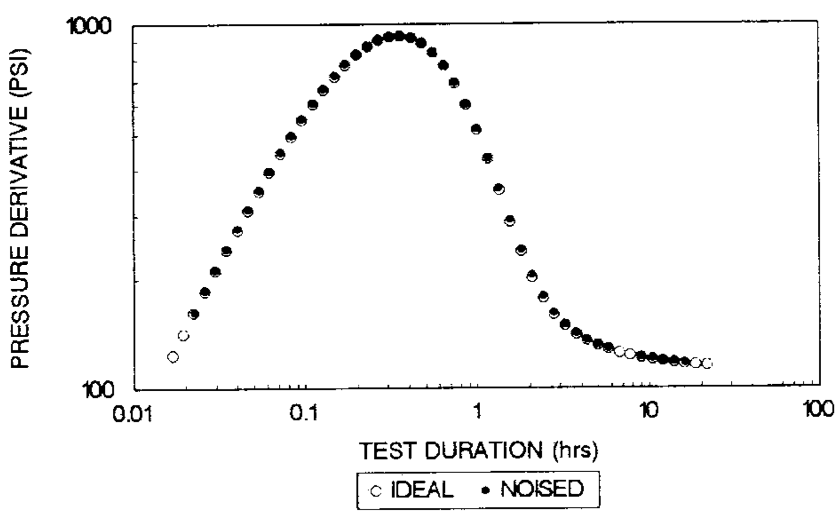


FIGURE 17: SMOOTHING BEFORE DIFFERENTIATION

USING ARITHMATIC AVERAGE (5 POINTS)

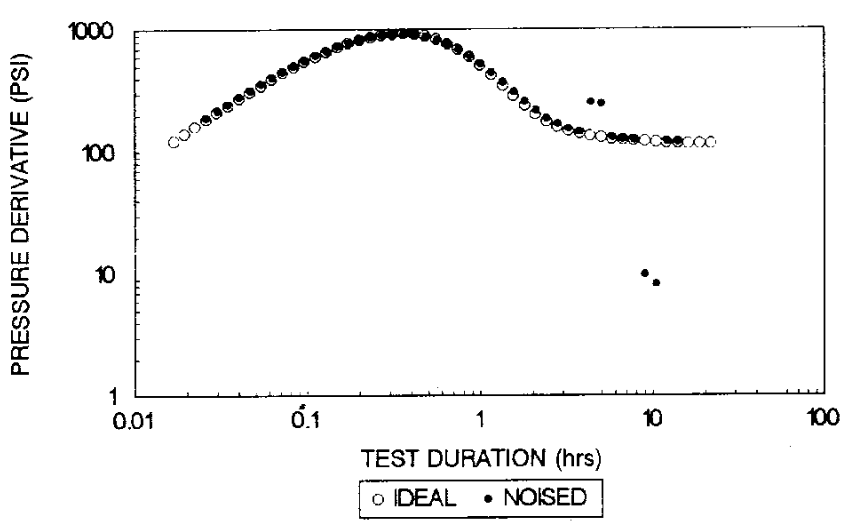


FIGURE 18: SMOOTHING BEFORE DIFFERENTIATION

USING DIRTY POINT FILTERING (h=0.01)

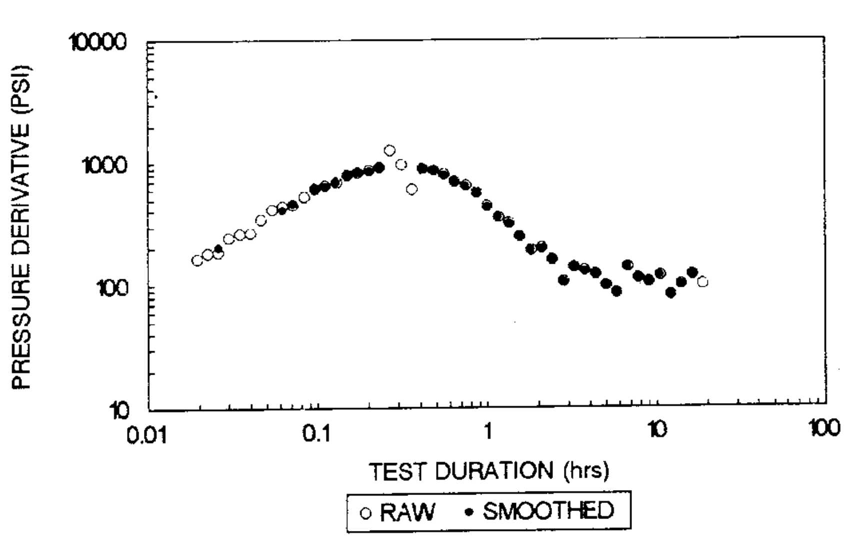


FIGURE 19: SMOOTHING BEFORE DIFFERENTIATION

USING ARITHMATIC AVERAGE (5 POINTS)

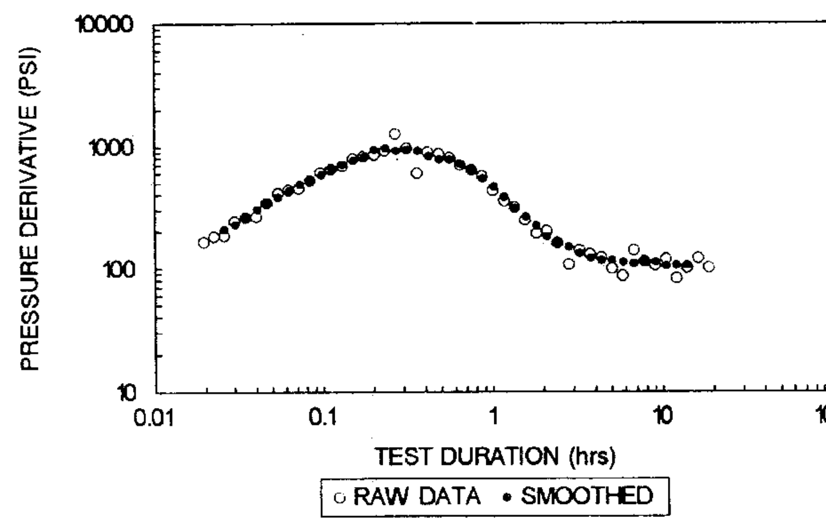


FIGURE 20: SMOOTHING BEFORE DIFFERENTIATION

USING NON-ADJASCENT POINTS (L=0.2)

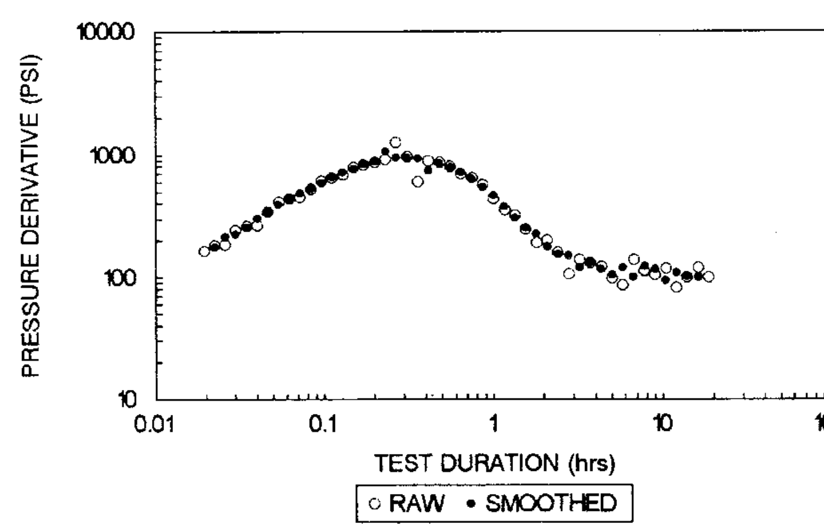


FIGURE 21: SMOOTHING AFTER DIFFERENTIATION

USING DIRTY POINT FILTERING (h=0.1)

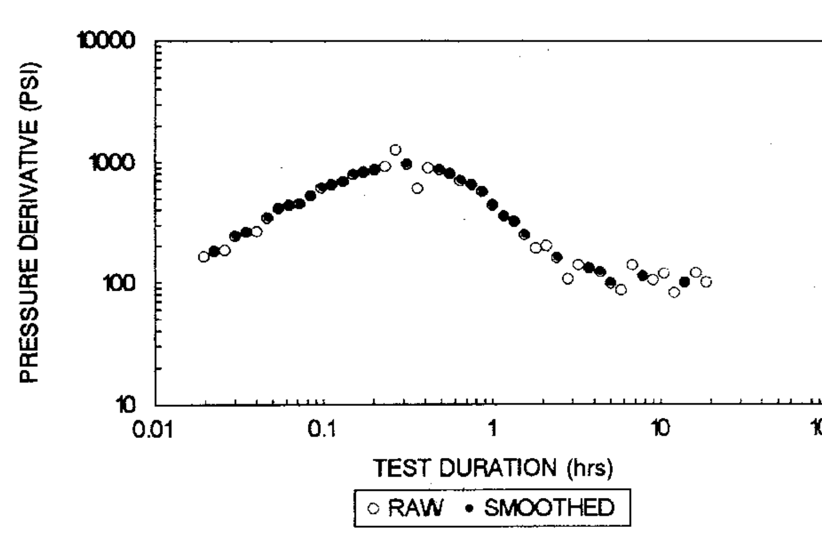
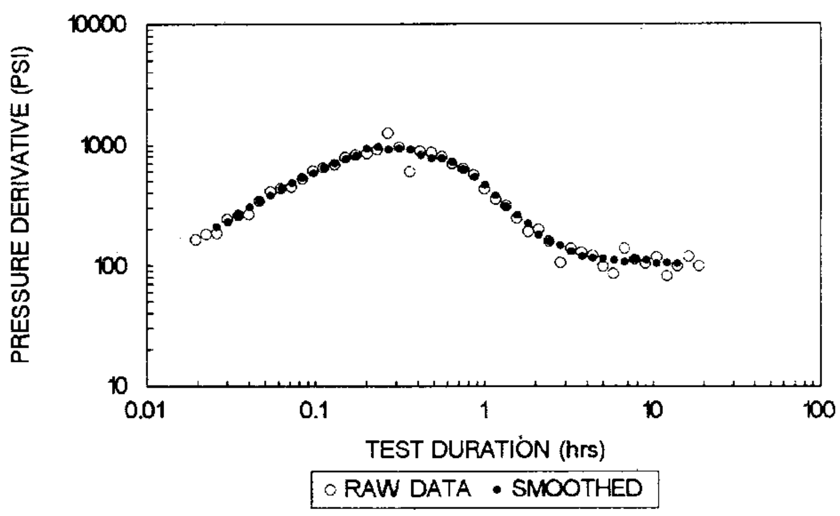


FIGURE 22: SMOOTHING AFTER DIFFERENTIATION

USING ARITHMATIC AVERAGE (5 POINTS)



SUBSCRIPTS

avg	average
d	dimensionless (pressure, radius, time)
o	oil
t	total
w	wellbore

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APPENDIX A

Fluid Properties:

$\beta o = 1.21$ bbl/stb
$C_t = 8.72E{-}06$ Psi^{-1}
$\mu = 0.92$ cp

Reservoir Properties:

$\phi = 0.21$
h=23 ft
k=77.1 md
$P_i = 6009$ Psi

Wellbore characteristics:

S= 6.09
C= 1.541E-02 bbl/psi
$r_w = 0.401$ ft

Using above data the following relationships were obtained;

S= 9.09
$C_d = 5.615\ C /(2\ C_t\ h\ r_w^2) = 2033$
$t_d = .000264\ K\ t / (C_t\ r_w^2) = t / 1.3914E{-}05$
$P_{wd} = Kh\ (P_i - P_{wf}) / (141.2\ q\ ) = (6009 - P_{wf}) / 221.6$
$t\ d(P) / dt = 221.6\ t_d\ dP_{wd}/dtd$
where $P = P_i - Pwf$