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Slope Based Mathematical Model for Curve Analysis

Khalid Amin Khan[1]

ABSTRACT

Many problems of applied physics involve analysis and interpretation of curves corresponding to some physical parameters. The most common analysis is interpreting the trend of a curve on the basis of its varying slope. A slope based mathenatical model for curve analysis is developed in the present study which finds vast application in geophysical problems, such as electrical resistivity and seismic refraction. In these problems the model automatically interprets, from the field curves, the number of layers and the position of interfaces between successive layers.

INTRODUCTION

In physical problems a curve indicates the variation of a parameter as a function of another parameter; usually time or space. This curve is the representative of a stream of data, for the concerned parameters. An appreciable change in the trend of the curve occurs when critical changes in the magnitude of parameters occur. In geophysical problems such changes are usually encountered at the interfaces between successive layers.

The mathematical model identifies the critical points at which the trend of a curve is changing, by analyzing the slope variations and splits the curve into separate layers.

MECHANISM

To analyze a curve the slopes between consecutive data points are calculated. A comparison between successive slopes is made by passing the curve through a window. The window picks two consecutive slopes at a time by taking the first slope as its base value and comparing the second relative to this base value. If the second slope lies within the limits of the window then no appreciable change in the trend has been encountered but if it lies outside the window limits then a new layer has been encountered. In this way successive layers are compared relatively. This mechanism is shown in Figure-1A where at point L the second slope lies within the window range therefore the layer continues, but at point M the second slope lies outside the window therefore a new layer has been encountered. The window is shown in Figure-1B where A, the first slope, is taken as the base and the limits are taken as A+B and A-B. The factor B is some percentage value of A which is added and subtracted to A to get the two limits. Thus the second slope is compared relative to the percentage limits of the first slope.

The mathematical treatment is as follows:

Let X_i, Y_i be the data coordinates with i varying from 1 to n.

where n is the number of data points.

The consecutive slopes S_h between the data points are given as,

$$S_h = \frac{dy_h}{dx_h} = \frac{y_{h+1} - y_h}{x_{h+1} - x_h} \quad \Bigg| \quad h \text{ varies from 1 to } n\text{-1}$$

Let $r=1$, $m=r$, $Lx_m = x_m$, $Ly_m = y_m$

$$\Longrightarrow \quad Lx_1 = x_1, \quad Ly_1 = y_1$$

where Lx_m, Ly_m are the coordinates of curve layering points and Lx_1, Ly_1 is taken as first layering point.

The following loop compares successive slopes by passing them through the window with limits A+B and A-B.

$$A = S_{j-1}$$
$$B = K * A \quad \Bigg| \quad j \text{ varies from 2 to } n\text{-1}.$$
$$C = S_j$$

[1] Oil and Gas Development Corporation, Islamabad.

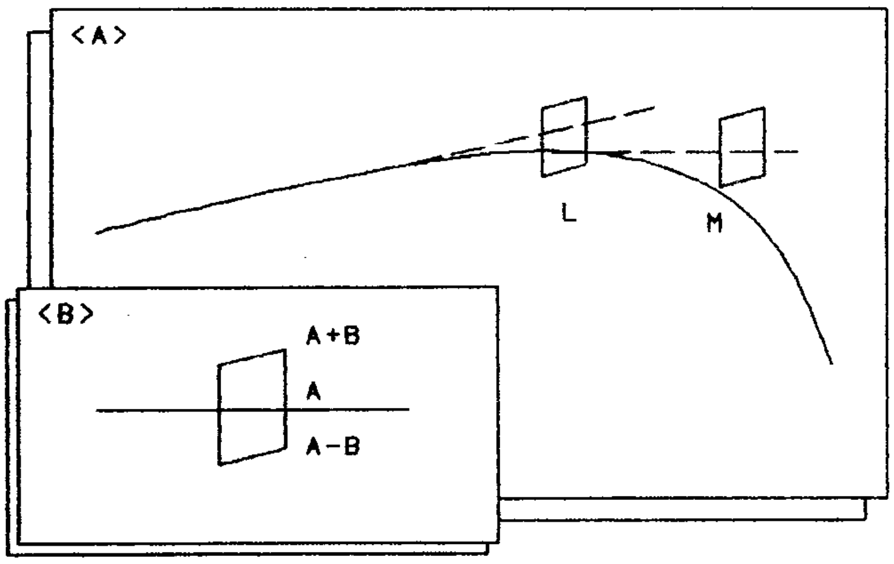


Figure 1– (A) Mechanism of curve analysis; (B) Window limits.

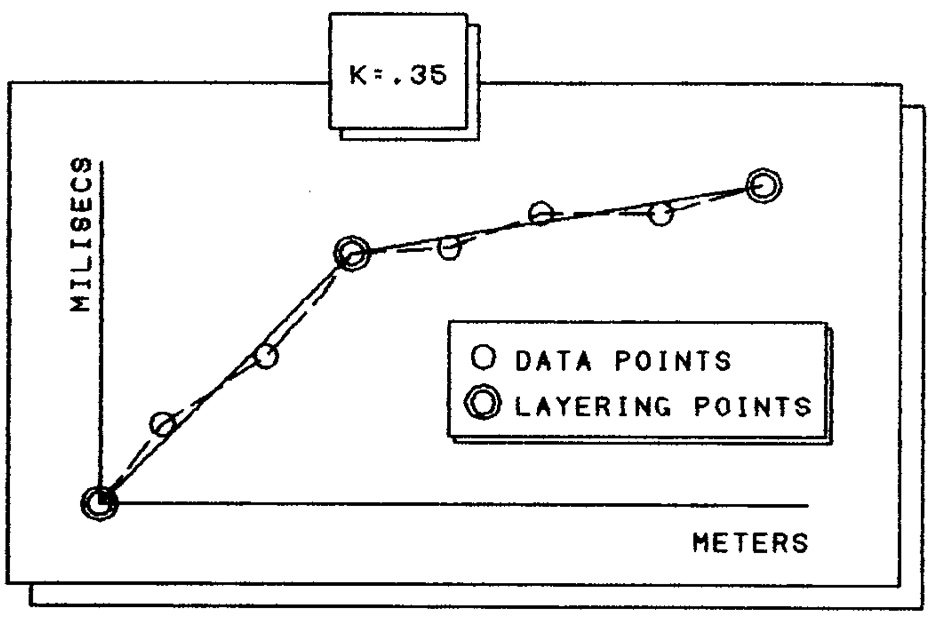
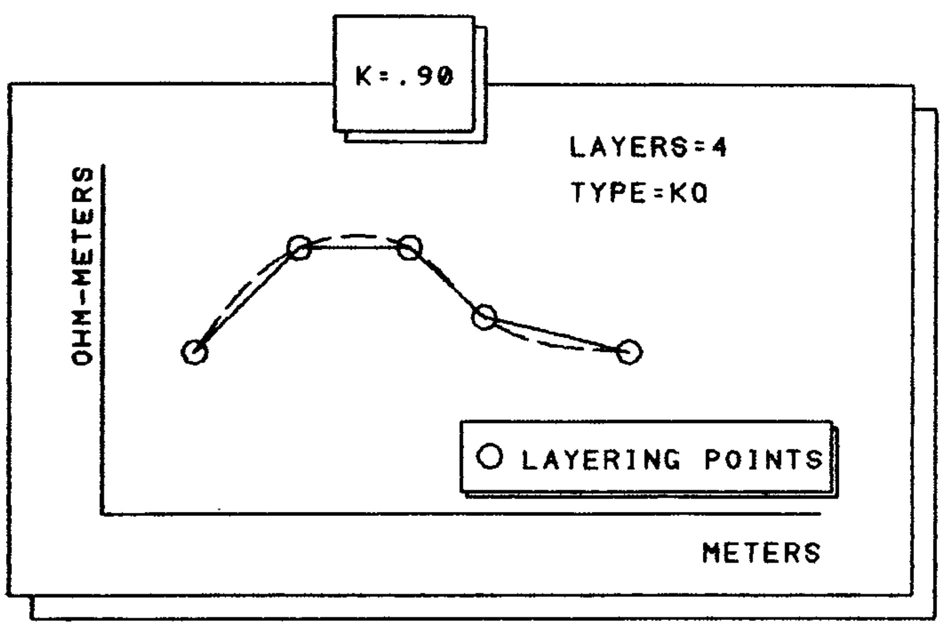


Figure 2– Application of Slope Model on electrical resistivity data.



Fifure 3– Application of Slope Model on refraction data.

Whenever a slope lying outside the window limits exits one of the following "if conditions" is activated and a new layering point is encountered.

if $C \not> 0$ and $(A\text{-}B) \not< C \not< (A+B)$	$m = r + 1,\ r = m,$
if $C \not< 0$ and $(A\text{-}B) \not> C \not> (A+B)$	$Lx_m = x_j,\ Ly_m = y_j$

Finally the last data point is taken as the last layering point.

$$Lx_{m+1} = x_n\ ,\ Ly_{m+1} = y_n$$

The factor K decides the width of the window as a percentage value of A. Its value is critical and varies for each application.

The above mathematics is modulated into a computer program (Program No. 1) written in BASIC.

For a given set of data points the range of other parameters is given below:

$n =$ number of data points
n-1 = number of slopes
n-1 $\geq m =$ number of layers, and
m+1 = number of layering points.

The mathematical model can be applied to a number of geophysical problems such as electrical resistivity (Kunetz, 1966) and seismic refraction (Johnson, 1976). Adding the lines given in program No.2 to program No.1 and keeping the value of K between 0.85 to 0.95 resistivity curves can be analyzed. The resulting program in addition to identifying the number and position of interfaces also gives the type of resistivity curve. An example has been given in Figure 2.

Similarly it has been determined that keeping the value of K between 0.35 to 0.40 refraction TX-graphs can be analyzed, as shown in Figure 3. This model has been used in siesmic refraction analysis software (SeiRA)[1] for processing real field data.

In addition to the above applications the model can be successively applied to a number of problems ranging from physics to statistical economics. For each application the data can be generated through program No.4 and the value of K can be determined by using program No.3.

REFERENCES

Johnson, S.M., 1976, Interpretation of split refraction data in terms of plane dipping layers: Geophysics, v.41, p.418-424.

Kunetz, G., 1966, Principles of direct current resistivity prospecting, Geopublication Associates.

[1] Application software for digiSEIZ-VRS was awarded 1st professional prize at 4th All Pakistan Software Competition and Exhibition.

PROGRAM NO.1

```
1   ' (C) KHALID AMIN KHAN.
10  P=40:DIM X(P),Y(P),S(P):LX(P):LY(P) :K=.35 '###
    OPTIONAL,USED FOR DATA INPUT
20  E$=".DAT":INPUT "ENTER INPUT FILE NAME: " ,F$
30  OPEN F$+E$ FOR INPUT AS #1
40  INPUT #1,N
50  FOR I = 1 TO N:INPUT #1,X(I),Y(I):NEXT
60  CLOSE #1: '————————————
80  FOR H = 1 TO N-1 '#### SLOPE BASED CURVE
90  S(H)=(Y(H+1)-Y(H))/(X(H+1)-X(H)) '#### ANALYSIS
    PROGRAM,
100 NEXT          '#### USED FOR DETERMINING
110 L=1           '#### THE CURVE BREAKUP
120 LX(1)=X(1):LY(1)=Y(1)          '#### POINTS.
130 FOR J = 2 TO N-1
140 A=S(J-1)
150 B=K*A
160 C=S(J)
170 IF CO AND C  (A-B) OR CO AND C  (A+B) THEN
    L=L+1:LX(L)=X(J):LY(L)=Y(J)
180 IF C AND C  (A-B) OR C AND C  (A+B) THEN
    L=L+1:LX(L)=X(J):LY(L)=Y(J)
190 NEXT
200 LX(L+1)=X(N):LY(L+1)=Y(N): '————————————
220 PRINT "LAYERS:";L:FOR T=1 TO L+1:PRINT
    LX(T),LY(T):NEXT
```

PROGRAM NO. 2

```
220 PRINT "LAYERS:";L:FOR T=1 TO L+1:PRINT
    LX(T),LY(T):NEXT
230 T=0      '#### LINES ADDED TO PROGRAM #1
240 FOR R = 2 TO L           '#### FOR ANALYZING
250 M = LY(R)               '#### VES CURVES
260 N = LY(R+1)
270 IF NM THEN T=(T*10)+1
280 IF N THEN T=(T*10)+2
290 NEXT
300 PRINT "TYPE=";:T$=STR$(T):F=LEN(T$):F=F-2:W=0
310 FOR V=2 TO F+1
320 T=T-W:E=10^(F+1-V):W=T/E:W=INT(W)
330 IF W=12 THEN L$="K"
340 IF W=21 THEN L$="H"
350 IF W=22 THEN L$="Q"
360 IF W=11 THEN L$="A"
370 PRINT L$;
380 E=10^(F+2-V):W=T/E:W=INT(W):W=W*E
390 NEXT
```

PROGRAM NO. 3

```
1   ' (C) KHALID AMIN KHAN.
10  P=40:DIM LX(P),LY(P)  '#### TEST PROGRAM TO
    DETERMINE THE VALUE OF K
20  DIM X(P),Y(P),S(P)    '#### FOR A PARTICULAR
    APPLICATION
40  E$=".DAT":INPUT "ENTER INPUT FILE NAME: " ,F$
50  OPEN F$+E$ FOR INPUT AS #1
60  INPUT #1,N
80  FOR I = 1 TO N:INPUT #1,X(I),Y(I):NEXT
100 CLOSE #1
120 FOR H = 1 TO N-1     '————————————
130 S(H)=(Y(H+1)-Y(H))/(X(H+1)-X(H))
140 NEXT
150 L=1
160 LX(1)=X(1):LY(1)=Y(1)
170 FOR J = 2 TO N-1
180 A=S(J-1)
190 B=K*A
200 C=S(J)
210 IF CO AND C  (A-B) OR CO AND C  (A+B) THEN
    L=L+1:LX(L)=X(J):LY(L)=Y(J)
220 IF C AND C  (A-B) OR C AND C  (A+B) THEN
    L=L+1:LX(L)=X(J):LY(L)=Y(J)
230 NEXT
240 LX(L+1)=X(N):LY(L+1)=Y(N)   '————————
260 CLS:PRINT "K: ";K
270 PRINT "DATA POINTS":FOR T=1 TO N:PRINT
    X(T),Y(T):NEXT
280 LOCATE 2,30:PRINT "SLOPES":FOR T=1 TO
    N-1:LOCATE T+2,30:PRINT S(T):NEXT
290 LOCATE 2,50:PRINT "LAYERS:";L:FOR T=1 TO
    L+1:LOCATE T+2,50:PRINT LX(T),LY(T):NEXT
300 Y$=INKEY$:IF Y$="" THEN GOTO 300
310 SCREEN 9:CLS
320 LINE (X(1)*4,Y(1)*4)-(X(1)*4,Y(1)*4):FOR T=1 TO
    N:LINE -(X(T)*4,Y(T)*4),4:NEXT
330 Y$=INKEY$:IF Y$="" THEN GOTO 330
340 CIRCLE (LX(1)*4,LY(1)*4),4,14:
    FOR T=2 TO L+1:LINE-(LX(T)*4,LY(T)*4),1:
    CIRCLE(LX(T)*4,LY(T)*4),4,14:NEXT
350 LOCATE 1,55:PRINT "(C) Khalid Amin Khan 1989"
360 LOCATE 20,1:PRINT:PRINT "K: ";K:INPUT "NEW K:
    ",K:GOTO 150
```

PROGRAM NO.4

```
1   ' (C) KHALID AMIN KHAN.
10  INPUT "ENTER OUTPUT FILE NAME: ",F$ '####
    TEST PROGRAM TO GENERATE AND
20  E$=".dat"                '#### SAVE DATA POINTS
30  OPEN F$+E$ FOR OUTPUT AS #1
40  INPUT "Number of data points: ",N
50  PRINT #1,N
60  FOR T = 1 TO N
70  PRINT "Enter (";T;") data points (X,Y):
    ";:INPUT"",X,Y
80  WRITE #1,X,Y
90  NEXT
100 CLOSE #1
```